Bi-Annual Research Journal "BALOCHISTAN REVIEW" ISSN 1810-2174 Balochistan Study Centre, University of Balochistan, Quetta (Pakistan) VOL. 44, 2019 (SPECIAL EDTION)

Time Series Forecasting with SARIMA Model of Water Outflow of Indus at Terbela River in Pakistan

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Abstract

We have made forecasting of the water outflow of Indus at Terbela river using the Seasonal Autoregressive Integrated Moving Average Model. Given Time series has showed seasonality pattern and it is incorporate in the ARIMA model. The best model is selected among many candidate models for forecasting the water outflow. The forecast values are tabulated up to 2020 and graphically shown. The policy maker may use these predicted values for their purposes.

Keywords: Stationarity, Autoregressive, Moving average, Autoregressive Moving average, Seasonality, Mean Square Error

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1. Introduction

The present study is based on time series forecasting of river flows data of Indus at Terbala river by applying adequate methods like Box-Jenkins method (Sarjinder Singh, 2003). Autoregressive Integrated Moving Average (ARIMA) is widely used model for the analysis of stochastic time series models (Jam, 2013) , (Hipel, 1994) model.

To apply the ARIMA it is assumed that time series is linear and follows a particular known statistical distribution (Hipel, 1994), (Adhikari K. & R.K., 2013) Moving Average (MA) (Seymour, Brockwell, & Davis, 1997) and Autoregressive Moving Average (ARMA) (Mills, 2015) models. For seasonal time series forecasting, Box and Jenkins (Klose, Pircher, & Sharma, 2004) had proposed a quite successful variation of ARIMA model, viz. the Seasonal ARIMA (Elganainy & Eldwer, 2018). ARIMA is most famous and widely used model in time series analysis due to its flexibility to represent several varieties of time series with simplicity as well as the associated Box-Jenkins methodology (Since & Model, 2008), (Naveena, Singh, Rathod, & Singh, 2017).

1.1 **Background of Study:**

After Box-Jenkins, neural systems are taken place as an efficient technique. In 1940s, the systems of neural nets have its birthplace (McCulloch, 1943) and (Hebb, 1949) which has been seen into systems of basic processing method that can be demonstrate/model neurological action and acquiring inside these systems, individually.

1.2 Problem Statement:

The economical procedures based on the forecasting issues for slight over the improvement of methods. Statistical and Econometrical methods are generally used the information as a part of overseeing generation frameworks. These methods have additionally revealed common application in an variety of other issue ranges. In financial matters, there are numerous issues which require the utilization of large panel of time series.

1.3 Significance/Justification of Study:

The recent technique is the way toward utilizing likelihood to attempt to forecast the probability of specific occasions happening in the future.

a) Operations management

b) Marketing

c) Finance & Risk management

d) Economics

e) Industrial Process Control Demography:

f) forecast of population

1.4 Objectives of Study

We focus on the real-world problem in the Pakistan Tarbela River. The objects of this study to fit the Model, identification, estimation and forecasting as well as forecasting to use SARIMA.

1.5 Research Questions:

1. How to identify the model?

2. How to estimate model?

3. How to forecast the model?

1.6 Limitation of Study:

There are some limitations regarding this study because its finding may not be generalized for the entire Pakistan Tarbela river flows due to the listed below reasons:

It adopts recent techniques

It is restricted to a specific data

The status of its finding may vary from time to time

2. Literature Review

(Chatfield, 1996) a model is rarely pre-specified but rather is typically formulated in an iterative, collaborative way using the given time-series data. Empirically, it established that the more complicated models tend to give a better fit but forecasts.

(Jones & Smart, 2005) demonstrated that Autoregressive modelling is used to explore the internal structure of long-term records of nitrate concentration for five karst springs in the Mendip Hills and found that there exists significant short term positive autocorrelation at three of the five springs due to the availability of sufficient nitrate within the soil store to sustain concentrations in winter revive for several months.

(Cai, Lye, & Khan, 2009) have been provided flood warnings to the residents living along the various sections of the Humber River Basin, the Water Resources Management Division (WRMD) of the Department of

Environment and Conservation has calculated flow forecasts for this basin over the years by means of numerous rainfall-runoff models.

(Rbunaru & Cescu, 2013) provided the time series data formed in both dynamic and time series which differ from other data sets which are arranged according to the time variable. The important and essential component of time series are seasonal fluctuations along with trend, cyclical and random oscillations are seasonal fluctuations. They provided the suitable method for the analysis of seasonal fluctuations.

(Valipour, Banihabib, Mahmood, & Behbahani, 2013) presented in their present study of forecasting the inflow of Dez dam reservoir by using ARMA and ARIMA models and increased the number of parameters to enhance the accuracy of forecasting. They compared these models with the static and dynamic ANNs.

(Jam, 2013) has tried to explore the adequate Auto Regressive Integrated Moving Average model by through Box-Jenkins methodology to predict the area of mangoes in Pakistan. In this present study time series data from 1961 to 2009 has been used for forecasting of area of mangoes.

(Merkuryeva & Kornevs, 2014) presented the methods for flood forecasting and simulation which are applied to a river flood analysis and risk prediction. They applied different models for different water flow forecasting and river simulation.

It was the time when the creation of (Box , 1976), ARIMA models, furthermore called Box–Jenkins models, have been an extremely understood kind of time course of action models used as a bit of hydro-logical forecasting the values. For the recent appliances, see for instance (Maria Castellano , 2004).

3. Research Methodology

Let $Y_1, Y_2, Y_3, ..., Y_t, ...$ be the elements of time series and the mean and variance at time *t* are given by

$$\mu_t = E[Y_t]$$

$$\sigma_t^2 = E\left[(Y_t - \mu_t)^2\right]$$

The covariance of Y_t, Y_s by

$$Cov(Y_t, Y_s) = E\left[(Y_t - \mu_t)(Y_s - \mu_s)\right]$$

Definition 3.1

If the mean, variance and covariances are independent of time the time series is called stationary, mathematically,

$$\mu_t = \mu, \qquad t = 1, 2, 3, ...$$

$$\sigma_t = \sigma, \qquad t = 1, 2, 3, ...$$

$$\lambda_{t,s} = \lambda_{t-s}, \qquad t \neq s$$

Definition 3.2

In time series autocorrelation is used instead of covariances and it may be defined as

$$\rho_{\upsilon =} \frac{\lambda_{t,t+\upsilon}}{\lambda_0} = \frac{\lambda_\upsilon}{\lambda_0} = \frac{E\left[(Y_t - \mu)(Y_{t+\upsilon} - \mu)\right]}{E\left[(Y_t - \mu)^2\right]}$$

Autoregressive Process AR(1) 3.3

Let ε_t be white noise and Y_t is an AR(1) process if

$$Y_t = \alpha Y_{t-1} + \varepsilon_t \quad where\left(|\alpha \prec 1|\right)$$

$$Y_t = \varepsilon_t + \alpha\left(\alpha Y_{t-2} + \varepsilon_{t-1}\right)$$

$$Y_t = \varepsilon_t + \alpha\varepsilon_{t-1} + \alpha^2 Y_{t-2}$$

$$Y_t = \varepsilon_t + \alpha\varepsilon_{t-1} + \alpha^2\left(\alpha Y_{t-3} + \varepsilon_{t-2}\right)$$

$$Y_t = \varepsilon_t + \alpha\varepsilon_{t-1} + \alpha^2\varepsilon_{t-2} + \alpha^3 Y_{t-3}$$

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$$Y_t = \varepsilon_t + \alpha\varepsilon_{t-1} + \alpha^2\varepsilon_{t-2} + \alpha^3\varepsilon_{t-3} + \ldots$$

$E\left[Y_t\right] = 0$; It is independent of time.

Also, the autocorrelation is

$$\lambda_k = E\left[Y_t, Y_{t+k}\right]$$

$$\lambda_k = E\left[\sum_{i=0}^{\infty}\alpha^i\varepsilon_{t-i}\right]\left[\sum_{i=0}^{\infty}\alpha^i\varepsilon_{t-i}\right]$$

$$\lambda_k = \sum_{i=0}^{\infty}\alpha^i\alpha^{k+i}\sigma_\varepsilon^2$$

$$\lambda_k = \alpha^{k+i}\sigma_\varepsilon^2\sum_{i=0}^{Inf}\alpha^{2i}$$

$$\lambda_k = \sigma_\varepsilon^2\frac{\alpha^k}{1-\alpha^2}\rho_k$$ where

$$\rho_k = \frac{\upsilon_k}{\upsilon_0}$$

$$\rho_k = \alpha^k k = 0,1,2,3,\ldots$$

$\rho_k = \alpha^{|k|} k = 0, \pm 1, \pm 2, \pm 3, \ldots$ which is independent of time. Thus AR(1) is stationary process.

Notation of Lag Operators 3.4

Consider the time series $Y_1, Y_2, Y_3, \ldots, Y_t$ then we define the lag operator represented by L as given below:

$$LY_t = Y_{t-1} \qquad if\ \alpha(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \alpha_3 L^3 - \ldots - \alpha_p L^p$$

Then we may define AR(1) process as follow:

$$Y_t = \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + \alpha_3 Y_{t-3} + \ldots + \alpha_p Y_{t-p} + \varepsilon_t$$ where ε_t is

white noise. So, using lag operator, it may be written as

$$Y_t = \alpha_1 L Y_t + \alpha_2 L^2 Y_t + \alpha_3 L^3 Y_t + \ldots + \alpha_p L^p Y_t + \varepsilon_t$$

$$\left(1 - \alpha_1 L - \alpha_2 L^2 - \alpha_3 L^3 - \ldots - \alpha_p L^p\right) Y_t = \varepsilon_t$$

$$\alpha(L) Y_t = \varepsilon_t \quad \text{where } |\alpha| \prec 1$$

Autoregressive Process AR(2) 3.5

AR(2) process may be written as

$$Y_t = \xi_1 Y_{t-1} + \xi_2 Y_{t-2} + \varepsilon_t$$

Using lag operators, we have

$$\left(1 - \xi_1 L - \xi_2 L^2\right) Y_t = \varepsilon_t$$

Also the process may be write as

$$Y_t = \zeta(L) \varepsilon_t$$

$$Y_t = \left(1 + \zeta_1 L + \zeta_2 L^2 + \zeta_3 L^3 + \ldots\right) \varepsilon_t$$

Where

$$\left(1 - \xi_1 L - \xi_2 L^2\right)^{-1} = \left(1 + \zeta_1 L + \zeta_2 L^2 + \zeta_3 L^3 + \ldots\right)$$

$$\left(1 - \xi_1 L - \xi_2 L^2\right)\left(1 + \zeta_1 L + \zeta_2 L^2 + \zeta_3 L^3 + \ldots\right) \equiv 1$$

Equating coefficients, we have

$$L^1: \qquad -\xi_1 + \zeta_1 = 0 \qquad \Rightarrow \qquad \zeta_1 = \xi_1$$

$$L^2: \qquad -\xi_2 + \xi_1 \zeta_1 + \zeta_2 = 0 \qquad \Rightarrow \qquad \zeta_1 = \xi_1^2 + \xi_1$$

$$L^3: \qquad -\xi_2 + \xi_1^2\zeta_1 + \zeta_2 = 0 \quad \Rightarrow \quad \zeta_1 = \xi_1^3 + 2\xi_1\xi_2$$

$$L^j: \qquad \zeta_j = \xi_1\zeta_{j-1} + \xi_2\zeta_{j-2}$$

All the weights can be determined recursively.

Autoregressive Process AR(p) 3.6

AR(p) is defined as

$$Y_t - \xi_1 Y_{t-1} - \xi_2 Y_{t-2} - \dots - \xi_p Y_{t-p} = \varepsilon_t$$

$$\left(1 - \xi_1 L - \xi_2 L^2 - \xi_3 L^3 - \dots - \xi_p L^p\right) Y_t = \varepsilon_t$$

$$\Phi(L) Y_t = \varepsilon_t \quad \text{where}$$

$$\Phi(L) = \left(1 - \xi_1 L - \xi_2 L^2 - \xi_3 L^3 - \dots - \xi_p L^p\right)$$

With the following condition of stationarity that can be set out as follows by writing

$$\Phi(L) = (1 - h_1 L)(1 - h_2 L)(1 - h_3 L)\dots(1 - h_p L)$$

$$|h_i| \prec 1 \quad for \quad i = 1, 2, 3, \dots, p$$

Alternatively, we may write

h_i^{-1} all lie outside the unit circle.

The autocorrelation will follow a difference equation of the form

$$\Phi(L)\rho_k = 0 \quad for \quad k = 1, 2, 3, \dots$$

Which has the solution in the form

$$\rho_k = A_1 h_1^k + A_2 h_2^k + A_3 h_3^k + \dots + A_p h_p^k$$

Moving Average Process MA(1) 3.7

Here an MA(1) process can be defined as

$$Y_t = \varepsilon_t + \theta\varepsilon_{t-1}$$

Where ε_t is the white noise

$$E[Y_t] = 0$$

$$Var[Y_t] = E\left[\left(\varepsilon_t + \theta\varepsilon_{t-1}\right)^2\right]$$

$$Var[Y_t] = E\left[\varepsilon_t^2 + \theta^2 E\left(\varepsilon_t^2\right)\right]$$

$$Var[Y_t] = \left(1 + \theta^2\right)\sigma_\varepsilon^2$$

$$\lambda_1 = E[Y_t Y_{t-1}]$$

$$\lambda_1 = E\left[\left(\varepsilon_t + \theta\varepsilon_{t-1}\right)\left(\varepsilon_{t-1} + \theta\varepsilon_{t-2}\right)\right]$$

$$\lambda_1 = E\left[\varepsilon_{t-1}^2\right]$$

$$\lambda_1 = \theta\sigma_\varepsilon^2$$

So, we have

$$\rho_1 = \frac{\theta}{1 + \theta^2}$$

$$\lambda_2 = E\left[\left(\varepsilon_t + \theta\varepsilon_{t-1}\right)\left(\varepsilon_{t-2} + \theta\varepsilon_{t-3}\right)\right]$$

$$\lambda_2 = 0$$

Generally, $\lambda_j \geq 0 \; for \; j \succ 2$. So Moving Average process MA(1) is stationary irrespective of the value of θ.

Moving Average MA(q) process can be defined in the following fashion.

$$Y_t = \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \theta_3\varepsilon_{t-3} + \ldots + \theta_q\varepsilon_{t-q}$$

$$E\left[Y_t\right]=0$$

$$Var\left[Y_t\right]=\left(1+\theta_1^2+\theta_2^2+\theta_3^2+\ldots+\theta_q^2\right)\sigma_\varepsilon^2$$

$$\lambda_k=Cov\left[Y_tY_{t-k}\right]$$

$$\lambda_k=E\left[\begin{matrix}\left(\begin{matrix}\varepsilon_t+\theta_1\varepsilon_{t-1}+\theta_2\varepsilon_{t-2}+\theta_3\varepsilon_{t-3}+\ldots+\theta_k\varepsilon_{t-k}\\+\theta_{k+1}\varepsilon_{t-k-1}+\theta_{k+2}\varepsilon_{t-k-2}+\theta_{k+3}\varepsilon_{t-k-3}+\ldots+\theta_q\varepsilon_{t-q}\end{matrix}\right)\\\times\left(\varepsilon_{t-k}+\theta_1\varepsilon_{t-k-1}+\theta_2\varepsilon_{t-k-2}+\theta_3\varepsilon_{t-k-3}+\ldots+\theta_{q-k}\varepsilon_{t-q}+\ldots\right)\end{matrix}\right]$$

$$\lambda_k=\left(\theta_k+\theta_{k+1}\theta_1+\theta_{k+2}\theta_2+\ldots+\theta_q\theta_{q-k}\right)\sigma_\varepsilon^2 \text{ and}$$

$$\rho_k=\frac{\lambda_k}{Var\left[Y_t\right]}$$

Also, for moving average process to be stationary regardless of the values of the $\theta^{,s}$.

$$\rho_k=\begin{cases}\dfrac{\sum_{i=0}^{n-k}\left(\theta_i,\theta_{i+k}\right)}{\left(1+\theta_1^2+\theta_2^2+\theta_3^2+\ldots+\theta_q^2\right)} & , \quad k\le q\\ 0 & , \quad k\succ q\end{cases}$$

Autoregressive Moving Average ARMA(p,q) Process 3.8

It is mixture of AR(p) and MA(q) processes of order p,q and it is represented as ARMA(p,q) if

$$Y_t=\xi_1Y_{t-1}+\xi_2Y_{t-2}+\xi_3Y_{t-3}+\ldots+\varepsilon_t+\theta_1\varepsilon_{t-1}+\theta_2\varepsilon_{t-2}+\theta_3\varepsilon_{t-3}+\ldots+\theta_q\varepsilon_{t-q}$$

$$\left(1-\zeta_1L-\zeta_2L^2-\zeta_3L^3-\ldots-\zeta_pL^p\right)Y_t=\left(1+\theta_1L+\theta_2L^2+\theta_3L^3+\ldots+\theta_qL^q\right)\varepsilon_t$$

Or $\quad \Phi\left(L\right)Y_t=\Theta\left(L\right)\varepsilon_t$

Where the polynomials Φ and Θ of degree p and q respectively in *L*.

Box-Jenkins Methodology 3.9

Box-Jenkins methodology is very famous and widely used method of univariate time series analysis, which is a hybirid of the AR and MA models

$$Y_t = \xi_1 Y_{t-1} + \xi_2 Y_{t-2} + \xi_3 Y_{t-3} + \ldots + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \theta_3 \varepsilon_{t-3} + \ldots + \theta_q \varepsilon_{t-q}$$

where the terms in the equation have the same meaning as given for the AR and MA model.

To apply the Box-Jenkins method, it is assumed that the time series is stationary. If the time series data is non-stationary then Box and Jenkins recommend differencing one or more times to achieve stationarity which produces an ARIMA model, where the "I" stands for "Integrated".

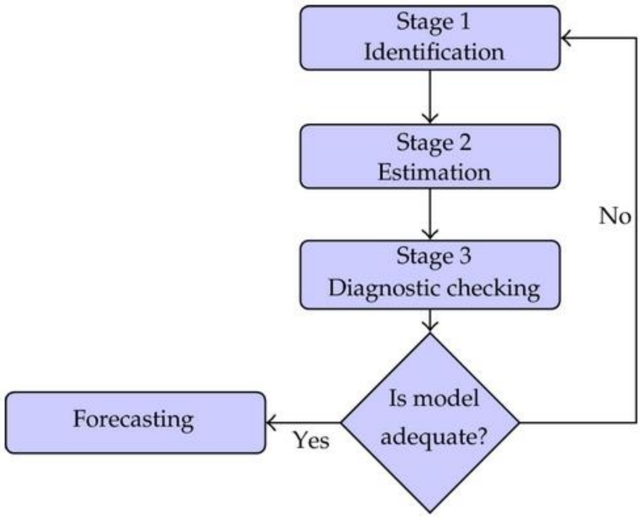
To include the seasonal autoregressive and seasonal moving average terms, we extends the Box-Jenkins models which may complicate the notations and mathematics of the model.

Box-Jenkins model commonly includes difference operators, autoregressive terms, moving average terms, seasonal autoregressive terms, seasonal difference operators, , and seasonal moving average terms.

The four stages of methodology are:

- Identification
- Estimation
- Diagnostic check on Model adequacy
- Forecasting

These steps may be shown by a logic flow diagram as below:



Box-Jenkins Methodology stage of Identification 3.9.1

For the implementation of the Box-Jenkins model, the initial first step is to determine if the series is stationary.

Only the partial autocorrelation function computed from sample is generally not helpful for identifying the order of the moving average process. Here are the basic guideline are listed below for the identification of AR(p) and MA(q) processes.

Process	*ACF*	*PACF*
White noise	All $\rho_s = 0, s \neq 0$.	All $\varphi_s = 0$.
AR($p = P$)	Decay toward zero. Coefficients may oscillate (decay is geometric if $P = 1$).	Spikes at s = P. All $\varphi_s = 0$ for $s > P$.
MA($q = Q$)	Spike at $s = Q$ and $\rho_s = 0$ $\forall\, s \neq Q$.	Decay toward zero (either direct or oscillatory).
ARMA ($p = P$, $q = Q$)	Decay (either direct or oscillatory) beginning at lag Q.	Decay (either direct or oscillatory) beginning after lag P.

NOTE: ACF = autocorrelation function, AR = autoregressive, ARMA = autoregressive moving average, MA = moving average, PACF = partial autocorrelation function.

Box-Jenkins Methodology stage of Estimation 3.9.2

The step for the implementation of Box-Jenkins methodology is the estimating the parameters. Therefore, the parameter estimation is accomplished using sophisticated and high-quality software program that successfully implement Box-Jenkins models.

Box-Jenkins Methodology stage of Diagnostics 3.9.3

When some candidate models are listed then next step is to check diagnostics for Box-Jenkins models. It means that the error term as assumed to follow the assumptions for a stationary univariate process, That is residuals should be white noise, that is having a constant mean and variance. If the residuals satisfy these assumptions, then it is expected that Box-Jenkins model is a good model for the data and will forecast the time series better.

Box-Jenkins Methodology stage of Forecasting 3.9.4

When best is selected then it will be used for forecasting.

Seasonal ARIMA Model 3.9.5

If there exists the seasonal trend after k period of then we try to remove the seasonality from the series and produce a modified time series which may not be seasonal. After that we apply the Box-Jenkins methodology of ARIMA model. Mathematically, let v_t be the nonseasonal time series, then the proposed the seasonal ARIMA (SARIMA) model filter as

$$\Phi_k\left(L^k\right)\left(1-L^k\right)^D Y_t = \Theta_k\left(L^k\right)v_t$$

Where

$$\Phi_k\left(L^k\right)=1-\omega_{1k}L^k-\omega_{2k}L^{2k}-\omega_{3k}L^{3k}-...-\omega_{Pk}L^{Pk}$$

$$\Theta_k\left(L^k\right)=1-\tau_{1k}L^k-\tau_{2k}L^{2k}-\tau_{3k}L^{3k}-...-\tau_{Qk}L^{Qk}$$

Also v_t is the ARIMA(p,d,q) which is the estimated using the ARIMA model as follow

$$\Phi(L)(1-L)^d v_t = \Theta(L)\varepsilon_t$$

Putting for v_t , we have

$$\Phi(L^k)(1-L^k)^D \Phi_k(L^k)(1-L^k)^D = \Theta(L)\Theta_k(L^k)\varepsilon_t$$

Which is SARIMA.

3.1 Research Design:

Appropriate quantitative techniques will be applied for the finding of this study. The method will be compare to find the adequate and appropriate technique to forecast the values of the water outflow data.

3.2 Data Collection and analysis

Secondary time series data of water outflow is used from 2015 to 2019 recorded on daily basis. Appropriate and adequate statistical method have been applied using R, a statistical software.

4. Results and Discussion

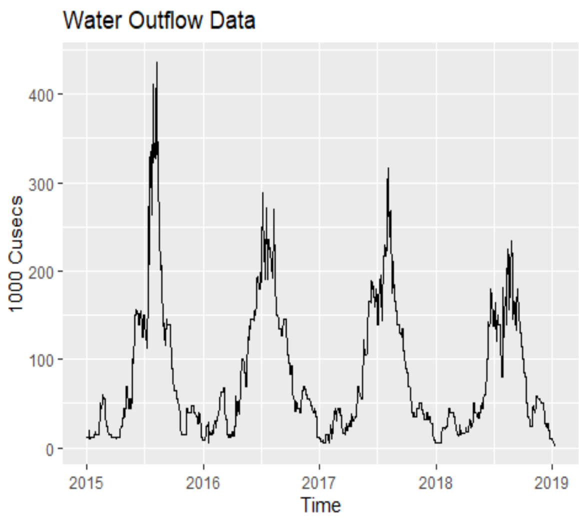


Figure 4.1:- Time series plot of Water Outflow data

From figure 4.1, it is shown that there is seasonality in the data as within a year it may be repeated in a similar way. It looks like a non-stationary time series as the mean and variance are not same with respect to time. We may note that there is gradual decrease in water outflow. For more elaboration we have other plots as given below.

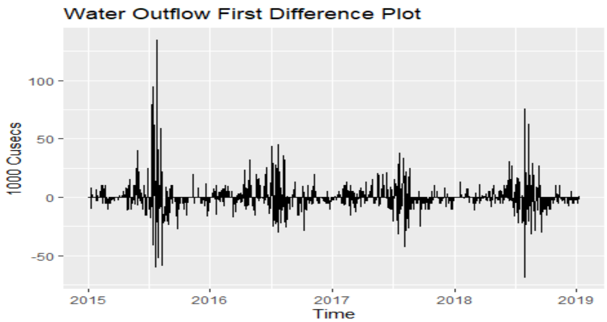


Figure 4.2:- The First Difference Plot of Water Outflow

From figure 4.2, it is shown the mean of the given series becomes the constant with respect to time after taking the first difference of the series. It can also be seen that there is a seasonal trend the series and it can be further explore as below:

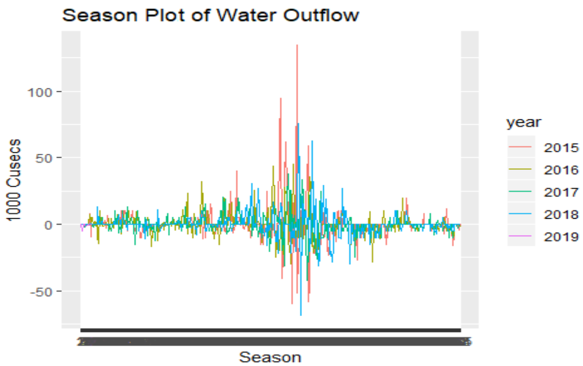


Figure 4.3:- Seasonal Plot of the Water outflow

From figure 4.3, The seasonal plot of the time series represents that there is almost same pattern during all the year which emphasis that there is seasonal trend the time series of the plot of the water outflow. It can be further explore by displaying the season subplot of Water Outflow data.

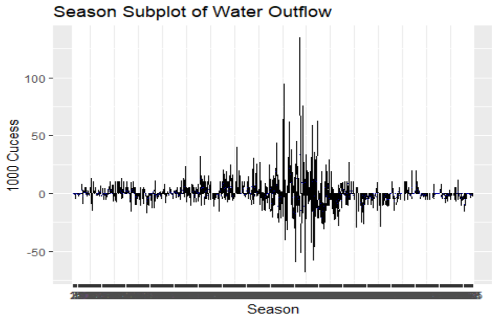


Figure 4.4:- Season Subplot of the Water Outflow data

From figure 4.4, The season subplot of the time series signifies that there is roughly same pattern during all the year which focus on seasonal trend the time series of the plot of the water outflow. This suggest that the given time series should be decompose.

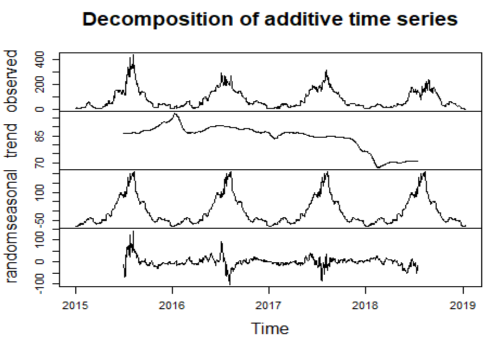


Figure 4.5:- Decomposition of the Water Outflow time series

From figure 4.5, Obviously, there is seasonal trend in the given time series of Water Outflow, it also shows that there is a decreasing trend.

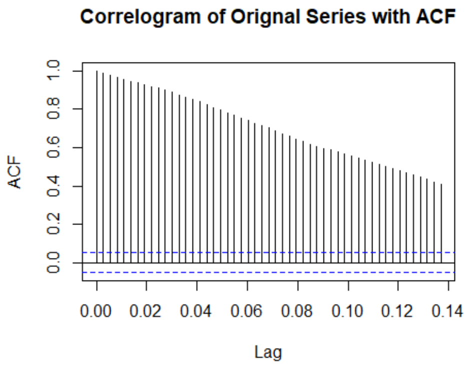


Figure 4.6:- Correlogram of the Water Outflow series with ACF

figure 4.6, it is shown that the process is memory driven process.

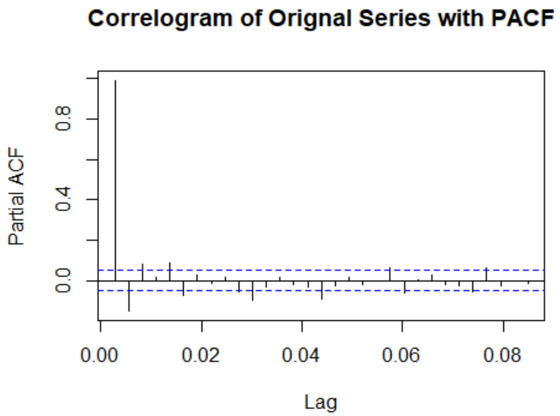


Figure 4.7:- Correlogram of the Water Outflow series with PACF

From figure 4.7, it is shown that the process is also moving average

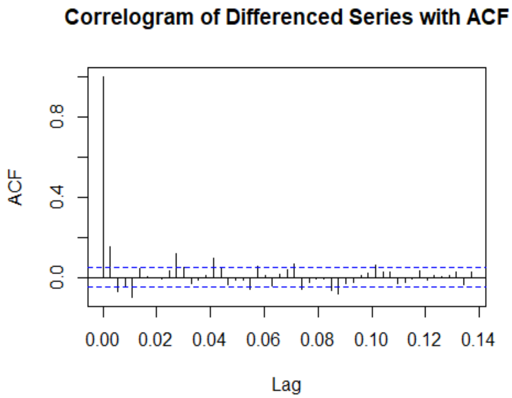


Figure 4.8:- Correlogram of the difference of Water Outflow series with ACF

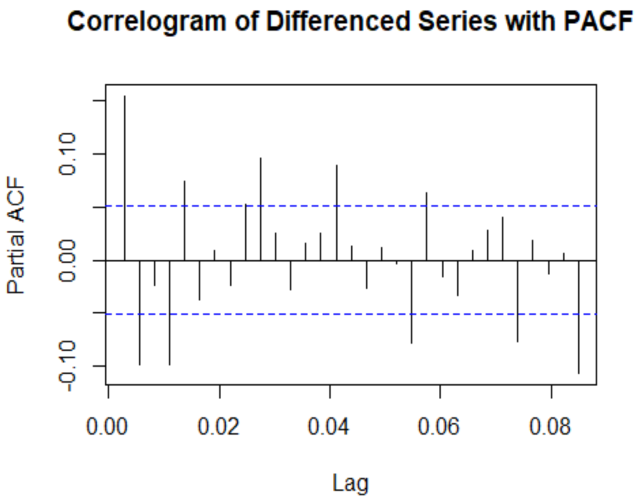


Figure 4.9:- Correlogram of the difference of Water Outflow series with PACF

Table 4.1 Candidate SARIMA Models

Model	AIC
ARIMA(0,1,0)(0,1,0)	8828.187
ARIMA(0,1,1)(0,1,0)	8795.579
ARIMA(0,1,2)(0,1,0)	8792.474
ARIMA(0,1,3)(0,1,0)	8791.784
ARIMA(0,1,4)(0,1,0)	8756.738
ARIMA(0,1,5)(0,1,0)	8757.509
ARIMA(1,1,0)(0,1,0)	8800.982
ARIMA(1,1,1)(0,1,0)	8789.731
ARIMA(1,1,3)(0,1,0)	8774.345
ARIMA(1,1,4)(0,1,0)	**8756.504**
ARIMA(2,1,0)(0,1,0)	8790.907
ARIMA(2,1,1)(0,1,0)	8770.703
ARIMA(3,1,0)(0,1,0)	8787.185
ARIMA(3,1,1)(0,1,0)	8772.593
ARIMA(4,1,0)(0,1,0)	8774.229
ARIMA(4,1,1)(0,1,0)	8765.531
ARIMA(5,1,0)(0,1,0)	8773.098

In the above table the candidate models for forecasting the Water Outflow data are tabulated, by studying the correlogram with respect to ACF and PACF.

Table 4.2 Summary of Seasonal ARIMA Model

IMA(1,1,4)(0,1,0)	(1)	.(1)	.(2)	.(3)	.(4)
mate	.741	444	)502	378	:082
	508	588	428	370	308

Table 4.3 Diagnostics of SARIMA

AIC	AICc	BIC	RMSE
8756.43	8756.5	8786.48	10.8925

Table 4.4 Ljung-Box test

	Q-statistic	df	p-value
SARIMA	353.11	28	0.0059

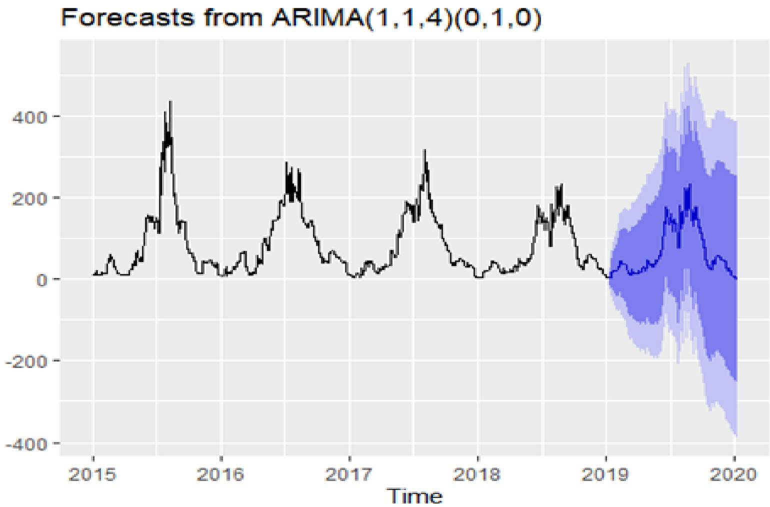


Figure 4.10:- Graphical Presentation of the Actual and Forecast

Table 4.5 Forecast Values Estimated through SARIMA Model

Date	Point Forecast	Date	Point Forecast	Date	Point Forecast
1/1/2019	2.87	1/24/2019	23.02	2/16/2019	33.02
1/2/2019	3.01	1/25/2019	30.02	2/17/2019	32.02
1/3/2019	2.98	1/26/2019	30.02	2/18/2019	29.32
1/4/2019	3.03	1/27/2019	33.02	2/19/2019	18.72
1/5/2019	15.92	1/28/2019	33.02	2/20/2019	18.72
1/6/2019	16.02	1/29/2019	40.02	2/21/2019	22.12
1/7/2019	16.02	1/30/2019	43.02	2/22/2019	17.22
1/8/2019	18.02	1/31/2019	43.02	2/23/2019	15.42
1/9/2019	20.02	2/1/2019	42.12	2/24/2019	14.22
1/10/2019	20.02	2/2/2019	38.02	2/25/2019	12.72
1/11/2019	20.02	2/3/2019	38.02	2/26/2019	12.72
1/12/2019	20.02	2/4/2019	38.02	2/27/2019	12.92
1/13/2019	20.02	2/5/2019	38.02	2/28/2019	12.32
1/14/2019	20.02	2/6/2019	38.02	3/1/2019	12.22
1/15/2019	20.02	2/7/2019	38.02	3/2/2019	11.82
1/16/2019	20.22	2/8/2019	38.02	3/3/2019	13.22
1/17/2019	19.82	2/9/2019	38.02	3/4/2019	24.42
1/18/2019	20.02	2/10/2019	38.02	3/5/2019	23.52
1/19/2019	20.02	2/11/2019	38.02	3/6/2019	18.82
1/20/2019	20.02	2/12/2019	38.02	3/7/2019	17.02

1/21/2019	23.02	2/13/2019	38.02	3/8/2019	15.72
1/22/2019	23.02	2/14/2019	33.02	3/9/2019	14.62
1/23/2019	23.02	2/15/2019	33.02	3/10/2019	13.72

Continued

Date	Point Forecast	Date	Point Forecast	Date	Point Forecast
3/11/2019	15.02	4/3/2019	25.52	4/26/2019	35.72
3/12/2019	14.52	4/4/2019	30.52	4/27/2019	35.42
3/13/2019	14.22	4/5/2019	28.02	4/28/2019	34.42
3/14/2019	14.22	4/6/2019	28.02	4/29/2019	33.42
3/15/2019	13.82	4/7/2019	28.02	4/30/2019	31.52
3/16/2019	14.32	4/8/2019	28.02	5/1/2019	32.52
3/17/2019	14.12	4/9/2019	28.02	5/2/2019	38.02
3/18/2019	14.42	4/10/2019	20.02	5/3/2019	38.02
3/19/2019	14.42	4/11/2019	23.02	5/4/2019	33.02
3/20/2019	14.92	4/12/2019	23.02	5/5/2019	33.02
3/21/2019	14.82	4/13/2019	23.02	5/6/2019	32.22
3/22/2019	15.52	4/14/2019	28.02	5/7/2019	38.02
3/23/2019	18.32	4/15/2019	38.02	5/8/2019	38.02
3/24/2019	20.42	4/16/2019	43.02	5/9/2019	38.02
3/25/2019	19.02	4/17/2019	48.02	5/10/2019	43.02
3/26/2019	16.02	4/18/2019	48.02	5/11/2019	48.02
3/27/2019	16.02	4/19/2019	43.02	5/12/2019	48.02
3/28/2019	16.02	4/20/2019	38.02	5/13/2019	48.02
3/29/2019	16.02	4/21/2019	38.02	5/14/2019	43.02
3/30/2019	16.02	4/22/2019	38.02	5/15/2019	43.02

3/31/2019	16.02	4/23/2019	38.02	5/16/2019	43.02
4/1/2019	16.02	4/24/2019	38.02	5/17/2019	43.02
4/2/2019	21.02	4/25/2019	38.02	5/18/2019	53.02

Continue

Date	Point Forecast	Date	Point Forecast	Date	Point Forecast
5/19/2019	58.02	6/11/2019	171.42	7/4/2019	138.02
5/20/2019	58.02	6/12/2019	170.92	7/5/2019	138.02
5/21/2019	58.02	6/13/2019	165.62	7/6/2019	138.02
5/22/2019	58.02	6/14/2019	157.12	7/7/2019	138.02
5/23/2019	58.02	6/15/2019	153.02	7/8/2019	138.02
5/24/2019	68.02	6/16/2019	151.82	7/9/2019	128.02
5/25/2019	68.02	6/17/2019	136.22	7/10/2019	106.62
5/26/2019	68.02	6/18/2019	140.52	7/11/2019	108.22
5/27/2019	78.02	6/19/2019	136.52	7/12/2019	98.02
5/28/2019	92.62	6/20/2019	135.22	7/13/2019	98.02
5/29/2019	98.02	6/21/2019	139.22	7/14/2019	78.02
5/30/2019	97.52	6/22/2019	144.62	7/15/2019	78.02
5/31/2019	98.52	6/23/2019	161.72	7/16/2019	78.02
6/1/2019	110.12	6/24/2019	160.92	7/17/2019	153.32
6/2/2019	140.92	6/25/2019	148.02	7/18/2019	180.02
6/3/2019	138.02	6/26/2019	143.22	7/19/2019	111.72
6/4/2019	138.02	6/27/2019	121.72	7/20/2019	133.02
6/5/2019	138.02	6/28/2019	118.62	7/21/2019	133.02

6/6/2019	136.12	6/29/2019	120.82	7/22/2019	148.02
6/7/2019	139.92	6/30/2019	136.52	7/23/2019	153.02
6/8/2019	167.12	7/1/2019	148.02	7/24/2019	163.02
6/9/2019	178.02	7/2/2019	148.02	7/25/2019	163.02
6/10/2019	178.02	7/3/2019	138.02	7/26/2019	168.02

Continue

Date	Point Forecast	Date	Point Forecast	Date	Point Forecast
7/27/2019	168.02	8/19/2019	143.22	9/11/2019	128.02
7/28/2019	148.02	8/20/2019	152.32	9/12/2019	128.02
7/29/2019	138.02	8/21/2019	160.82	9/13/2019	128.02
7/30/2019	200.42	8/22/2019	159.82	9/14/2019	118.02
7/31/2019	202.32	8/23/2019	163.62	9/15/2019	113.02
8/1/2019	195.42	8/24/2019	161.82	9/16/2019	113.02
8/2/2019	204.32	8/25/2019	151.92	9/17/2019	103.02
8/3/2019	223.32	8/26/2019	141.82	9/18/2019	98.02
8/4/2019	212.02	8/27/2019	130.92	9/19/2019	98.02
8/5/2019	180.62	8/28/2019	158.02	9/20/2019	98.02
8/6/2019	155.12	8/29/2019	158.02	9/21/2019	98.02
8/7/2019	157.02	8/30/2019	168.02	9/22/2019	98.02
8/8/2019	176.02	8/31/2019	168.02	9/23/2019	83.02
8/9/2019	179.12	9/1/2019	178.02	9/24/2019	83.02
8/10/2019	207.92	9/2/2019	178.02	9/25/2019	78.02
8/11/2019	204.12	9/3/2019	178.02	9/26/2019	78.02

8/12/2019	222.82	9/4/2019	148.02	9/27/2019	78.02
8/13/2019	228.22	9/5/2019	148.02	9/28/2019	68.02
8/14/2019	232.22	9/6/2019	148.02	9/29/2019	58.02
8/15/2019	230.72	9/7/2019	148.02	9/30/2019	48.02
8/16/2019	208.02	9/8/2019	135.62	10/1/2019	48.02
8/17/2019	187.52	9/9/2019	128.02	10/2/2019	38.02
8/18/2019	159.52	9/10/2019	128.02	10/3/2019	33.02

Continued

Date	Point Forecast	Date	Point Forecast	Date	Point Forecast
10/4/2019	33.02	11/2/2019	57.02	12/2/2019	21.02
10/5/2019	33.02	11/3/2019	57.02	12/3/2019	26.02
10/6/2019	33.02	11/4/2019	57.02	12/4/2019	26.02
10/7/2019	33.02	11/5/2019	54.02	12/5/2019	26.02
10/8/2019	33.02	11/6/2019	54.02	12/6/2019	26.02
10/9/2019	28.02	11/7/2019	53.02	12/7/2019	21.02
10/10/2019	28.02	11/8/2019	53.02	12/8/2019	21.02
10/11/2019	28.02	11/9/2019	53.02	12/9/2019	18.02
10/12/2019	23.02	11/10/2019	53.02	12/10/2019	18.02
10/13/2019	23.02	11/11/2019	53.02	12/11/2019	13.02
10/14/2019	23.02	11/12/2019	53.02	12/12/2019	13.02
10/15/2019	23.02	11/13/2019	48.02	12/13/2019	8.02
10/16/2019	23.02	11/14/2019	48.02	12/14/2019	8.02
10/17/2019	23.02	11/15/2019	48.02	12/15/2019	8.02

10/18/2019	33.02	11/16/2019	48.02	12/16/2019	8.02
10/19/2019	43.02	11/17/2019	48.02	12/17/2019	8.02
10/20/2019	43.02	11/18/2019	48.02	12/18/2019	8.02
10/21/2019	46.02	11/19/2019	48.02	12/19/2019	8.02
10/22/2019	46.02	11/20/2019	48.02	12/20/2019	8.02
10/23/2019	43.02	11/21/2019	43.02	12/21/2019	3.12
10/24/2019	38.02	11/22/2019	43.02	12/22/2019	3.02
10/25/2019	38.02	11/23/2019	43.02	12/23/2019	3.02
10/26/2019	38.02	11/24/2019	38.02	12/24/2019	3.02
10/27/2019	39.12	11/25/2019	33.02	12/25/2019	1.72
10/28/2019	43.02	11/26/2019	26.02	12/26/2019	1.02
10/29/2019	53.02	11/27/2019	26.02	12/27/2019	0.52
10/30/2019	57.02	11/28/2019	26.02	12/28/2019	1.02
10/31/2019	57.02	11/29/2019	24.92	12/29/2019	1.02
11/1/2019	57.02	11/30/2019	23.02	12/30/2019	1.02
10/27/2019	39.12	12/1/2019	23.02	12/31/2019	1.02

Date	Point Forecast	Date	Point Forecast	Date	Point Foreca
10/4/2019	33.02	11/2/2019	57.02	12/2/2019	21.02
10/5/2019	33.02	11/3/2019	57.02	12/3/2019	26.02
10/6/2019	33.02	11/4/2019	57.02	12/4/2019	26.02
10/7/2019	33.02	11/5/2019	54.02	12/5/2019	26.02
10/8/2019	33.02	11/6/2019	54.02	12/6/2019	26.02
10/9/2019	28.02	11/7/2019	53.02	12/7/2019	21.02
10/10/2019	28.02	11/8/2019	53.02	12/8/2019	21.02
10/11/2019	28.02	11/9/2019	53.02	12/9/2019	18.02
10/12/2019	23.02	11/10/2019	53.02	12/10/2019	18.02
10/13/2019	23.02	11/11/2019	53.02	12/11/2019	13.02
10/14/2019	23.02	11/12/2019	53.02	12/12/2019	13.02
10/15/2019	23.02	11/13/2019	48.02	12/13/2019	8.02
10/16/2019	23.02	11/14/2019	48.02	12/14/2019	8.02
10/17/2019	23.02	11/15/2019	48.02	12/15/2019	8.02
10/18/2019	33.02	11/16/2019	48.02	12/16/2019	8.02
10/19/2019	43.02	11/17/2019	48.02	12/17/2019	8.02
10/20/2019	43.02	11/18/2019	48.02	12/18/2019	8.02
10/21/2019	46.02	11/19/2019	48.02	12/19/2019	8.02
10/22/2019	46.02	11/20/2019	48.02	12/20/2019	8.02
10/23/2019	43.02	11/21/2019	43.02	12/21/2019	3.12
10/24/2019	38.02	11/22/2019	43.02	12/22/2019	3.02
10/25/2019	38.02	11/23/2019	43.02	12/23/2019	3.02
10/26/2019	38.02	11/24/2019	38.02	12/24/2019	3.02
10/27/2019	39.12	11/25/2019	33.02	12/25/2019	1.72
10/28/2019	43.02	11/26/2019	26.02	12/26/2019	1.02

5. Conclusions and Recommendations

Table 5.1 Conclusions and Recommendations

Forecasting Methods	**RMSE**
SARIMA Model	10.8925
Bayesian Approach	8087.4049
Non-parametric Method KNN	180.3049
ANN with 5 Hidden nodes	11.0876
ANN fit with (10,5) hidden nodes	**3.4394**

Generally, it has often been seen that the adequate selection of the order of the SARIMA model and the number of input, hidden and output neurons is very much crucial for the effective and successful prediction of the values. We have listed the Mean Square Error (MSE) for the comparison of the models.

From the Table 5.1 the minimum MSE is attained by Artificial Neural Network ANN with (10,5) hidden nodes and declared as the best one while on the change of the number of hidden nodes and layers it becomes the 11.0876 which is not the least. As Seasonal Autoregressive Integrated Moving Average (SARIMA) model has the MSE 10.8925 which the which is the 2nd least value. As SARIMA model is a parametric technique and has reasonable low MSE, therefore, we conclude it the best one model for the forecasting of the water outflow data. Also, if someone has excellent art of selection of the number of nodes and hidden layers, the ANN is also the adequate technique in this respect.

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